CHAPTER FIVE

Synthesis of Mass-Exchange Networks—An Algebraic Approach

The graphical pinch analysis presented in Chapter Three provides the designer with a very useful tool that represents the global transfer of mass from the waste streams to the MSAs and determines performance targets such as MOC of the MSAs. Notwithstanding the usefulness of the pinch diagram, it is subject to the accuracy problems associated with any graphical approach. This is particularly true when there is a wide range of operating compositions for the waste and the lean streams. In such cases, an algebraic method is recommended. This chapter presents an algebraic procedure which yields results that are equivalent to those provided by the graphical pinch analysis. In addition, this chapter describes a systematic technique for matching waste-lean pairs and configuring MENs that realize the MOC solutions.

5.1 The Composition-Interval Diagram "CID"

The CID is a useful tool for insuring thermodynamic feasibility of mass exchange. On this diagram, $N_{sp} + 1$ corresponding composition scales are generated. First, a composition scale, y, for the waste streams is established. Then, Eq. (3.5) is employed to create N_{sp} corresponding composition scales for the process MSAs. On the CID, each process stream is represented as a vertical arrow whose tail corresponds to its supply composition while its head represents its target composition. Next, horizontal lines are drawn at the heads and tails of the arrows. These horizontal lines define a series of composition intervals. The number of intervals

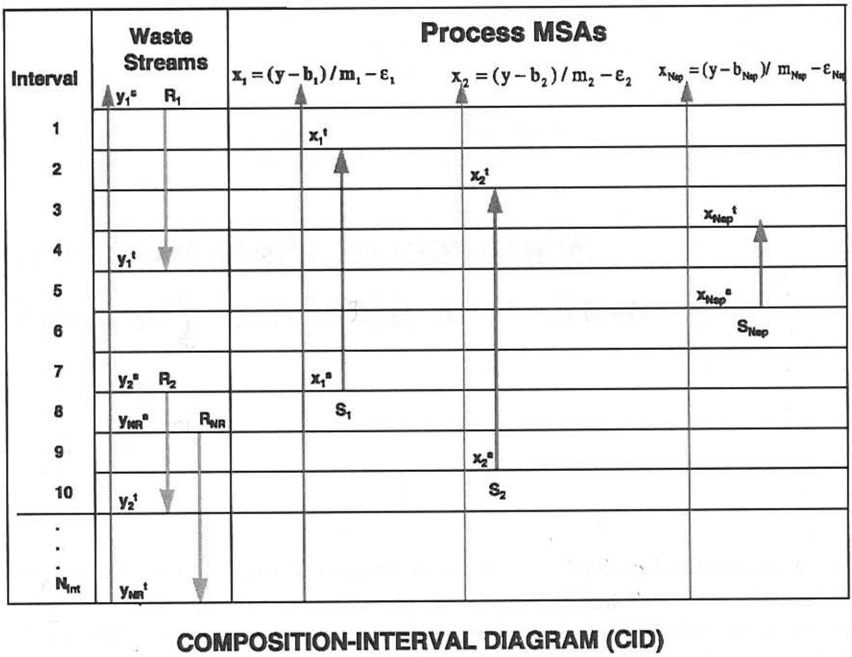

Figure 5.1 The composition interval diagram "CID".

is related to the number of process streams via

$$N_{int} \leq 2(N_R + N_{SP}) - 1, \tag{5.1}$$

with the equality applying in cases where no heads on tails coincide. The composition intervals are numbered from top to bottom in an ascending order. The index k will be used to designate an interval with $k = 1$ being the uppermost interval and $k = N_{int}$ being the lowermost interval. Figure 5.1 provides a schematic representation of the CID. Within any interval, it is thermodynamically feasible to transfer mass from the waste streams to the MSAs. It is also feasible to transfer mass from a waste stream in an interval k to any MSA which lies in an interval $\bar{k}$ below it (i.e., $\bar{k} \geq k$).

5.2 Table of Exchangeable Loads "TEL"

The objective of constructing a TEL is to determine the mass-exchange loads of the process streams in each composition interval. The exchangeable load of the

ith waste stream which passes through the kth interval is defined as

$$W_{i,k}^{R} = G_i(y_{k-1} - y_k), \tag{5.2}$$

where y_{k-1} and y_k are the waste-scale compositions of the transferrable species which respectively correspond to the top and the bottom lines defining the kth interval. On the other hand, the exchangeable load of the jth process MSA which passes through the kth interval is computed through the following expression

$$W_{j,k}^{S} = L_j^C(x_{j,k-1} - x_{j,k}), \tag{5.3}$$

where $x_{j,k-1}$ and $x_{j,k}$ are the compositions on the jth lean-composition scale which respectively correspond to the higher and lower horizontal lines bounding the kth interval. Clearly, if a stream does not pass through an interval, its load within that interval is zero.

Having determined the individual loads of all process streams for all composition intervals, one can also obtain the collective loads of the waste and the lean streams. The collective load of the waste streams within the kth interval is calculated by summing up the individual loads of the waste streams that pass through that interval, i.e.

$$W_k^R = \sum_{i \text{ passes through interval } k} W_{i,k}^R. \tag{5.4}$$

Similarly, the collective load of the lean streams within the kth interval is evaluated as follows:

$$W_k^S = \sum_{j \text{ passes through interval } k} W_{j,k}^S. \tag{5.5}$$

We are now in a position to incorporate material balance into the synthesis procedure with the objective of allocating the pinch point as well as evaluating excess capacity of process MSAs and load to be removed by external MSAs. These aspects are assessed through the mass-exchange cascade diagram.

5.3 Mass-Exchange Cascade Diagram

As has been mentioned earlier, the CID generates a number N_{int} of composition intervals. Within each interval, it is thermodynamically as well as technically feasible to transfer a certain mass of the key pollutant from a waste stream to a lean stream. Furthermore, it is feasible to pass mass from a waste stream in an interval to any lean stream in a lower interval. Hence, for the kth composition interval, one can write the following component material balance for the key pollutant:

$$W_k^R + \delta_{k-1} - W_k^S = \delta_k, \tag{5.6}$$

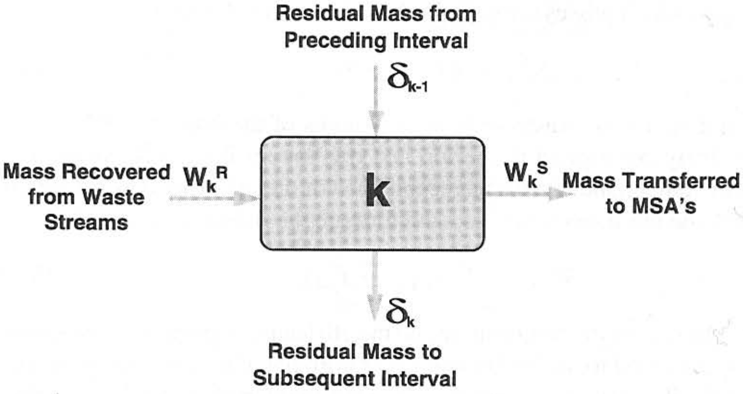

Figure 5.2 A pollutant material balance around a composition interval.

where δ_{k-1} and δ_k are the residual masses of the key pollutant entering and leaving the kth interval. Equation (5.6) indicates that the total mass input of the key component to the kth interval is due to collective load of the waste streams in that interval as well as the residual mass of the key component leaving the interval above it, δ_{k-1}. A total mass, W_k^S, of the key pollutant is transferred to the MSAs in the kth interval. Hence, a residual mass, δ_k, of the pollutant leaving the kth interval can be calculated via Eq. (5.6). This output residual also constitutes the influent residual to the subsequent interval. Figure 5.2 illustrates the component material balance for the key pollutant around the kth composition interval.

It is worth pointing out that δ_0 is zero since no waste streams exist above the first interval. In addition, thermodynamic feasibility is insured when all the δ_k's are nonnegative. Hence, a negative δ_k indicates that the capacity of the process lean streams at that level is greater than the load of the waste streams. The most negative δ_k corresponds to the excess capacity of the process MSAs in removing the pollutant. Therefore, this excess capacity of process MSAs should be reduced by lowering the flowrate and/or the outlet composition of one or more of the MSAs. After removing the excess capacity of MSAs, one can construct a revised TEL in which the flowrates and/or outlet compositions of the process MSAs have been adjusted. Consequently a revised cascade diagram can be generated. On the revised cascade diagram the location at which the residual mass is zero corresponds to the mass-exchange pinch composition. As expected, this location is the same as that with the most negative residual on the original cascade diagram. Since an overall material balance for the network must be realized, the residual mass leaving the lowest composition interval of the revised cascade diagram must be removed by external MSAs.