

Physical Chemistry I. practice

Gyula Samu

II.: Ideal gases

gysamu@mail.bme.hu

<http://oktatas.ch.bme.hu/oktatas/konyvek/fizkem/PysChemBSC1/Requirements.pdf>

http://oktatas.ch.bme.hu/oktatas/konyvek/fizkem/PysChemBSC1/Important_dates.pdf

Equations for the state changes of ideal gases

	W	Q	ΔU	ΔH	ΔS
Isobaric	$-nR\Delta T$	$nC_{m,p}\Delta T$	$nC_{m,v}\Delta T$	$nC_{m,p}\Delta T$	$nC_{m,p}\ln\frac{T_2}{T_1}$
Isochor	\emptyset	$nC_{m,v}\Delta T$	$nC_{m,v}\Delta T$	$nC_{m,p}\Delta T$	$nC_{m,v}\ln\frac{T_2}{T_1}$
Isothermal	$nRT\ln\frac{p_2}{p_1}$	$-nRT\ln\frac{p_2}{p_1}$	\emptyset	\emptyset	$-nR\ln\frac{p_2}{p_1}$
Ad. rev.	$nC_{m,v}\Delta T$	\emptyset	$nC_{m,v}\Delta T$	$nC_{m,p}\Delta T$	\emptyset

Adiabatic reversible:

Isothermal: $p_1/p_2 = V_2/V_1$

$$T_1/T_2 = (V_2/V_1)^{\kappa-1}$$

Isochor: $p_1/p_2 = T_1/T_2$

$$p_1/p_2 = (V_2/V_1)^\kappa$$

Isobaric: $V_1/V_2 = T_1/T_2$

$$T_1/T_2 = (p_2/p_1)^{\frac{1-\kappa}{\kappa}}$$

$$C_{m,p} - C_{m,v} = R$$

$$\kappa = \frac{C_{m,p}}{C_{m,v}}$$

Thermodynamic cycle

We perform a cycle process with 160 g of O_2 (ideal gas)

- From 20 °C and 0,1 MPa we compress it to 2 MPa in an adiabatic reversible process
- Then we heat it to 500 °C in an isochor process
- Then we expand it to 0,1 MPa in an isothermal process
- Finally we cool it to 20 °C in an isobaric process

What are W , Q , ΔU , ΔH , and ΔS

- in the four subprocesses?
- in the overall process?

$$(\kappa = 1,4)$$

Thermodynamic cycle

1. Ad. rev. $1 \rightarrow 2$

$$W_1 = nC_{m,v}(T_2 - T_1)$$

$$Q_1 = 0 \text{ J}$$

$$\Delta U_1 = W_1$$

$$\Delta H_1 = nC_{m,p}(T_2 - T_1)$$

$$\Delta S_1 = 0 \text{ J/K}$$

2. Isochor $2 \rightarrow 3$

$$W_2 = 0 \text{ J}$$

$$Q_2 = nC_{m,v}(T_3 - T_2)$$

$$\Delta U_2 = Q_2$$

$$\Delta H_2 = nC_{m,p}(T_3 - T_2)$$

$$\Delta S_2 = nC_{m,v} \ln(T_3/T_2)$$

3. Isothermal $3 \rightarrow 4$

$$W_3 = nRT_3 \ln(p_4/p_3)$$

$$Q_3 = -W_3$$

$$\Delta U_3 = 0 \text{ J}$$

$$\Delta H_3 = 0 \text{ J}$$

$$\Delta S_3 = -nR \ln(p_4/p_3)$$

Isobaric $4 \rightarrow 1$

$$W_4 = -nR(T_1 - T_4)$$

$$Q_4 = nC_{m,p}(T_1 - T_4)$$

$$\Delta U_4 = W_4 + Q_4$$

$$\Delta H_4 = Q_4$$

$$\Delta S_4 = nC_{m,p} \ln(T_1/T_4)$$

Thermodynamic cycle

$$n, C_{m,p}, C_{m,v}, T_2, p_3 ?$$

$$n = \frac{160g}{32g/mol} = 5 \text{ mol}$$

$$C_{m,p} = 1,4 C_{m,v}$$

$$\rightarrow 0,4 C_{m,v} = R \rightarrow C_{m,v} = \frac{5}{2}R, C_{m,p} = \frac{7}{2}R$$

$$\text{Ad. rev.: } T_2 = 293 \text{ K} \left(\frac{10^5 Pa}{2 \cdot 10^6 Pa} \right)^{\frac{1-1,4}{1,4}} \approx 690 \text{ K}$$

$$\text{Isochor: } p_3 = 2 \cdot 10^6 Pa \left(\frac{773 \text{ K}}{690 \text{ K}} \right) = 2,24 \cdot 10^6 Pa$$

Thermodynamic cycle

1. Ad. rev. $1 \rightarrow 2$

$$W_1 = nC_{m,v}(T_2 - T_1) = 41,3kJ$$

$$Q_1 = 0 J$$

$$\Delta U_1 = W_1$$

$$\Delta H_1 = nC_{m,p}(T_2 - T_1) = 57,82kJ$$

$$\Delta S_1 = 0 J/K$$

2. Isochor $2 \rightarrow 3$

$$W_2 = 0 J$$

$$Q_2 = nC_{m,v}(T_3 - T_2) = 8,6kJ$$

$$\Delta U_2 = Q_2$$

$$\Delta H_2 = nC_{m,p}(T_3 - T_2) = 12,0kJ$$

$$\Delta S_2 = nC_{m,v} \ln(T_3/T_2) = 11,8J/K$$

3. Isothermal $3 \rightarrow 4$

$$W_3 = nRT_3 \ln(p_4/p_3) = -99,9kJ$$

$$Q_3 = -W_3$$

$$\Delta U_3 = 0 J$$

$$\Delta H_3 = 0 J$$

$$\Delta S_3 = -nR \ln(p_4/p_3)$$

$$= 129,2J/K$$

4. Isobaric $4 \rightarrow 1$

$$W_4 = -nR(T_1 - T_4) = 19,9kJ$$

$$Q_4 = nC_{m,p}(T_1 - T_4) = -69,8kJ$$

$$\Delta U_4 = W_4 + Q_4$$

$$\Delta H_4 = Q_4$$

$$\Delta S_4 = nC_{m,p} \ln(T_1/T_4)$$

$$= -141,1J/K$$

$$\sum W = -34,8kJ, \sum Q = 34,8kJ$$

Sum of the functions of state is 0!

Different routes

We have 1 *mol* of argon (ideal gas) that is 25 °C and 10^5 Pa .

We heat and compress it to 100 °C and $5 \cdot 10^5 \text{ Pa}$.

$$C_{m,p} = 5/2R \text{ and } C_{m,v} = 3/2R.$$

What is the total W , Q , ΔU , and ΔH if

a)

- We first heat it to 100 °C on constant volume
- Then increase the pressure to $5 \cdot 10^5 \text{ Pa}$ on constant temperature?

Different routes

1. Isochor 1 \rightarrow 2

$$W_1 = 0 \text{ J}$$

$$Q_1 = nC_{m,v}(T_2 - T_1)$$

$$\Delta U_1 = Q_1$$

$$\Delta H_1 = nC_{m,p}(T_2 - T_1)$$

$$\Delta S_1 = nC_{m,v} \ln(T_2/T_1)$$

2. Isothermal 2 \rightarrow 3

$$W_2 = nRT_2 \ln(p_3/p_2)$$

$$Q_2 = -W_2$$

$$\Delta U_2 = 0 \text{ J}$$

$$\Delta H_2 = 0 \text{ J}$$

$$\Delta S_2 = -nR \ln(p_3/p_2)$$

Only p_2 is unknown

Isochor 1 \rightarrow 2

$$p_2 = 10^5 \text{ Pa} \left(\frac{373 \text{ K}}{298 \text{ K}} \right)$$

$$\cong 1,25 \cdot 10^5 \text{ Pa}$$

$$\sum W = 4294 \text{ J}$$

$$\sum Q = -3359 \text{ J}$$

$$\sum \Delta U = 935 \text{ J}$$

$$\sum \Delta H = 1559 \text{ J}$$

$$\sum \Delta S = -8,71 \text{ J/K}$$

Different routes

We have 1 *mol* of argon (ideal gas) that is 25 °C and 10^5 Pa .

We heat and compress it to 100 °C and $5 \cdot 10^5 \text{ Pa}$.

$$C_{m,p} = 5/2R \text{ and } C_{m,v} = 3/2R.$$

What is the total W , Q , ΔU , and ΔH if

b)

- We first heat it to 100 °C on constant pressure
- Then increase the pressure to $5 \cdot 10^5 \text{ Pa}$ on constant temperature?

Different routes

1. Isobaric 1 \rightarrow 2

$$W_1 = -nR(T_2 - T_1)$$

$$Q_1 = nC_{m,p}(T_2 - T_1)$$

$$\Delta U_1 = W_1 + Q_1$$

$$\Delta H_1 = Q_1$$

$$\Delta S_1 = nC_{m,p} \ln(T_2/T_1)$$

2. Isothermal 2 \rightarrow 3

$$W_2 = nRT_2 \ln(p_3/p_2)$$

$$Q_2 = -W_2$$

$$\Delta U_2 = 0 \text{ J}$$

$$\Delta H_2 = 0 \text{ J}$$

$$\Delta S_2 = -nR \ln(p_3/p_2)$$

Every variable is known

$$\sum W = 4367 \text{ J}$$

$$\sum Q = -3432 \text{ J}$$

$$\sum \Delta U = 935 \text{ J}$$

$$\sum \Delta H = 1559 \text{ J}$$

$$\sum \Delta S = -8,71 \text{ J/K}$$

Different routes

We have 1 m^3 argon (ideal gas) with 298 K temperature and 10^5 Pa pressure.

We compress it in an adiabatic reversible process, then expand it to its original volume in an isothermal process.

Its pressure becomes $2 \cdot 10^5 \text{ Pa}$.

$$C_{m,p} = 5/2R, C_{m,v} = 3/2R$$

What is the total change in entropy?

Different routes

In sum, it is an isochor process

$$\Delta S = nC_{m,v} \ln(T_3/T_1)$$

$$n = \frac{p_1 V_1}{RT_1} = 40,36 \text{ mol}$$

$$T_3 = \frac{p_3 V_3}{nR} = 596 \text{ K}$$

$$\Delta S = 348,88 \text{ J/K}$$

More argon

We have 1 m^3 argon (ideal gas) with 298 K temperature and 10^6 Pa pressure.

We expand it in an adiabatic reversible process to 2 m^3 .

$$C_{m,p} = 5/2R, C_{m,v} = 3/2R$$

What is the new T and p ?

What is the W , ΔU , and ΔH ?

More argon

$$\kappa = \frac{5}{3}$$

$$T_2 = 298 \text{ K} \left(\frac{1 \text{ m}^3}{2 \text{ m}^3} \right)^{\frac{2}{\kappa}} = 188 \text{ K}$$

$$p_2 = 10^6 \text{ Pa} \left(\frac{1 \text{ m}^3}{2 \text{ m}^3} \right)^{\frac{5}{\kappa}} = 3,15 \cdot 10^6 \text{ Pa}$$

$$n = \frac{10^6 \text{ Pa}}{R \cdot 298 \text{ K}} = 403,62 \text{ mol}$$

$$W = n C_{m,v} \Delta T = -554 \text{ kJ}$$

$$\Delta U = W$$

$$\Delta H = n C_{m,p} \Delta T = -923 \text{ kJ}$$